Int J Adv Manuf Technol (2015) 77:1943–1953
DOI 10.1007/s00170-014-6569-x

ORIGINAL ARTICLE

A robust decision-making approach for *p*-hub median location problems based on two-stage stochastic programming and mean-variance theory: a real case study

Taher Ahmadi · Hadi Karimi · Hamid Davoudpour · Seyed Abbas Hosseinijou

Received: 23 December 2011 / Accepted: 2 November 2014 / Published online: 25 November 2014

Abstract The stochastic location-allocation *p*-hub median problems are related to long-term decisions made in risky situations. Due to the importance of this type of problems in real-world applications, the authors were motivated to propose an approach to obtain more reliable policies in stochastic environments considering the decision makers' preferences. Therefore, a systematic approach to make robust decisions for the single location-allocation *p*-hub median problem based on mean-variance theory and two-stage stochastic programming was developed. The approach involves three main phases, namely location modeling, risk modeling, and decision making, each including several steps. In the first phase, the pertinent location-allocation model of the problem is developed in the form of a two-stage stochastic model based on its deterministic version. A risk measure, based on total cost function and mean-variance theory, is derived in the second phase. Furthermore, two heterogeneous terms of the risk measure have been normalized and an innovative procedure has been proposed to significantly improve the calculation efficiency. In the third phase, the Pareto solution is obtained, the frontier curve is depicted to determine the decision maker's risk aversion coefficient, and a robust policy is obtained through optimization based on decision makers' preferences. Finally, a case study of an automobile part distribution system with stochastic demand is described to further illustrate our risk management and analysis approach.

T. Ahmadi · H. Karimi · H. Davoudpour (✉) · S. A. Hosseinijou
Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran
e-mail: hamidp@aut.ac.ir

T. Ahmadi
Pars Khodro Automobile Manufacturer Company, Tehran, Iran

Keywords *p*-Hub location-allocation median problem · Two-stage stochastic programming · Risk analysis and management · Robust decision making · Risk aversion coefficient

1 Introduction

In recent years, due to the increasing need to produce and distribute goods and provide service just in time and more economically, hub location problems (HLPs) have been the focus of attention in facility planning literature. Hub location problems are usually considered as network location problems consisting of given pairs of nodes (origins/destinations) and the connecting edges between them. In this type of problems, it is assumed that goods or services are conveyed from the origin node to the destination through the hubs in order to fulfill the demand of each node. Some real-world examples for HLPs are transportation networks with passengers, mailing parcels or goods transported between the nodes, communication networks with data and information transmitted between the nodes, and emergency services that convey firefighting facilities or patients.

Due to different features and characteristics of real-world applications involving hubs, several models have been developed under the category of hub location problems. Based on the costs considered in the objective functions, two most frequent models introduced so far are the *p*-hub median and capacitated or uncapacitated hub location problems. In capacitated or uncapacitated hub location problems, setup costs for hubs and operation costs for flow of goods in the network are considered, but in the *p*-hub median problem, only the operation costs are being minimized. In capacitated hub location problems, there are extra limitations on the capacity that can flow through arcs and nodes. Based on the allocation of non-hub nodes (spokes) to hubs, there are two major classifications of hub location problems including single allocation and multiple allocation models. In single allocation models, every spoke is only connected to one hub. On the other hand, in multiple allocation models, each spoke can send and receive flow from more than one hub. Other than these two major approaches, in some models, flexible allocations between hubs and spoke are considered. The basic model that is studied in this paper is a single allocation uncapacitated hub location problem.

One important aspect of real-world applications is the uncertain environment that can affect important parameters of the model such as demand, prices, and distances. In order to make more realistic decisions in any problem, it is essential to consider the uncertain behavior of the determining parameters. In hub location problems, there have been few studies considering the uncertainty in modeling and solution procedure. The gap between the literature and the wide range of real-world applications of these models has motivated the authors to present a model with the assumption of uncertain demand data. In general, there are two possibilities towards considering uncertainty in problems: (i) the probability distribution associated with the uncertain data in the model is unknown, or (ii) uncertain parameters in the model follow a known probability distribution. The modeling approach related to the latter case is called stochastic programming. In this paper, a two-stage stochastic programming approach is presented that captures the uncertainty in demand of goods in the network.

Hub location problems are among the network design problems that are usually part of a long-term strategic decision-making problem. Designing and implementing hub and spoke networks occur before the network starts to operate; therefore, a reliable decision making is necessary to prevent failures and unexpected costs. Besides considering a stochastic programming approach, management of risks is another approach that helps to have reliable and efficient decisions in uncertain environments. Another contribution of this paper is adding a risk management measure to the reformulated stochastic hub location problem. The risk measure proposed is mean-variance risk measure, which is one of the basic measures firstly introduced in finance literature to manage the risk of portfolio selection problems by Markowitz [1].

To summarize, in this paper, an uncapacitated single allocation p-hub median (location) problem has been studied in uncertain conditions of demand flow. This problem is reformulated as a two-stage stochastic programming model to deal with uncertainties. Then, the mean-variance risk measure is added to the model to get more reliable solutions. Also, two heterogeneous terms of the risk measure have been normalized and an innovative procedure has been proposed to significantly improve the calculation efficiency. An illustrative case study in an automobile part distribution system has been indicated, where the model has been successfully applied to analyze and manage uncertainty conditions and to make effective and reliable decisions. The main contribution in this study is introducing a systematic approach, based on a basic financial risk measure (mean-variance) integrated in a two-stage stochastic framework on a hub location problem. This work could be an excellent starting point to pursue this valuable and useful concept for more developed and improved models and conditions.

The rest of the paper is organized as follows: In the next section, the relevant literature about p-hub location is reviewed. In Section 3, the characteristics of the considered problem will be described. The framework of the innovative systematic approach will be explained in Section 4. In Section 5, an automobile part distribution system in Iran is considered as a case study, and finally, in Section 6, the conclusions and further research suggestions are addressed.

2 Literature review

Research on standard hub location problems dates back to the seminal papers of O'Kelly [2, 3]. Since then, much work has been done in this area, all of which cannot be covered in this review. Readers are referred to Campbell et al. [4] for a comprehensive survey on models and solution methodologies developed until year 2002. For a survey of literature up to 2007, readers are referred to the study of Alumur et al. [5], and to review the literature from 2007 up to 2013, readers can refer to the study of Farahani et al. [6]. In the area of classic and general location problems, uncertainty has been studied vastly in the last three decades (see for instance [7–9]). For a more recent survey on stochastic location models, the reader can refer to the study of Snyder [10].

Few studies in the area of hub location problems have provided models and algorithms with addressing uncertain parameters. In some papers, it is assumed that uncertain parameters follow a known distribution model. For example, Mariano and Serra [11] suggested a queuing model for hub location problems in airline industry where the arrivals of planes follow a Poisson process (see also [12] as an extension of this study). In another study, Sim et al. [13] developed a stochastic model for p-hub median problem in package delivery networks and assumed that the travel time between the nodes follow normal distribution.

Most of the times, we cannot identify the probability distribution of uncertain parameters. One important approach in dealing with this situation is defining possible scenarios for the occurrences of uncertain parameter and deriving their corresponding probabilities based on historical data and experts' opinion. Then, stochastic programing methods can be applied to model these scenarios and their corresponding probabilities. Yang [14] studied an airfreight hub location problem with stochastic seasonal demand and developed a two-stage stochastic programming model for this problem. His aim was to decide on the location of the hubs at the first stage and to determine the routes at the second stage.

Contreras et al. [15] provided an uncapacitated multiple allocation hub location problem with stochastic demand and transportation costs which was formulated as a two-stage integer programming (see also [16, 17]). Our study is in line with this stream of research on p-hub median problems where there is no known probability distribution for uncertain parameter (demand between nodes), and we provide a two-stage stochastic model. In this model, the location of hubs is determined in the first stage and the allocation of demands between hubs is determined in the second stage.

It is shown that in many real-world problems using only a stochastic modeling approach without considering possible risks of the decisions made will not provide reliable results [18]. In the field of facility location, most of the previous studies were risk neutral and applying risk management tools has not been studied until recent years. Main difference among risk-related studies is the measure used to account for risk in the problem. The work of Wagner et al. [19] is one of the seminal papers in risk-related studies in location problems, in which mean-variance and value-at-risk (VaR) risk measures have been used to manage the risk in a simple uncapacitated facility location problem. Other recent papers are Wang and Watada [20], where VaR-based location models are studied under fuzzy random uncertainty, and Azad and Davoudpour [21], where a stochastic location-routing model involving VaR measure has been studied.

To the best of our knowledge, there are two papers in the literature addressing risk in decision making for stochastic hub location problems. Zhai et al. [22] developed a new two-stage stochastic model for HLP with stochastic demand. To consider risk in their problem, they formulated the objective function in a way that the total costs were bounded by a predetermined upper bound. Mohammadi et al. [23] developed a stochastic multi-objective hub covering location problem. The uncertain parameter is the transportation time between nodes of the network, and a factor accounting for the risk of deviation of the expected transportation time is included in their model. In our study, we have modeled the problem by including the well-known mean-variance risk measure and provided a framework to reliable and robust decision making.

3 Problem statement

A general communicative network consisting of demand points and their relationships is considered in the form of a connected graph $G=(N,A)$, in which p nodes should be selected as hubs and the rest are defined as non-hub nodes. Also, it is assumed that the connection among p-hub nodes creates a complete graph and each non-hub node should be connected to only one hub node (single allocation). Therefore, if $|N|=n$, then $|A| = \frac{p(p-1)}{2} + (n-p)$, where $|N|$ and $|A|$ are the number of node and edges in graph G, respectively. Furthermore, transshipment for a specific stochastic demand between each pair of non-hub nodes (which have not been connected to the same hub) as origin and destination should be done in three different stages: collection (from origin non-hub to hub), transfer (from hub to hub), and distribution (from hub to destination non-hub), as depicted in Fig. 1.

In this problem, the main purpose is to provide a systematic approach to make robust decisions in order to determine which nodes must be considered as hubs (location) and how to assign non-hub nodes to them (allocation) such that the summation of transportation and setup cost is minimized.

4 Methodology

In this section, a systematic procedure consisting of three main phases, namely location modeling, risk modeling, and decision making, is presented in order to obtain a robust policy for p-hub location problem in a stochastic situation (Fig. 2). Each phase of this procedure will be discussed in the following subsections.

4.1 Location modeling

4.1.1 Notations

In order to propose the mathematical programming, the required notations are defined as follows:

Sets	
N	A set of graph node as communicative network
Φ	The set of all possible scenarios for stochastic demand
Parameters	
c_{ij}	Transportation unit cost between nodes i and j per demand unit
h_{ij}	Deterministic demand or flow that must be transferred from origin i to destination j
p	The number of hubs that should be opened
f_k	The setup cost of opening a hub facility at potential node k
α	Discount factor for transportation cost between hubs
ϕ	A realized scenario, $\phi \in \Phi$
$p(\phi)$	The probability of occurring scenario ϕ, $\phi \in \Phi$
λ	Decision maker's risk aversion coefficient
Main decision variables	
x_j	A binary variable that takes on the value 1 if node j is a hub, and it is 0, otherwise
y_{ik}	A binary variable that takes on the value 1 if node i is located to hub at node k, otherwise 0
Auxiliary terms	
TC	Total cost consisting of setup and transportation cost
$F(x)$	Total setup cost
$Q(x,\xi)$	Total stochastic transportation cost
RM	Risk measure, defined based on mean-variance theory

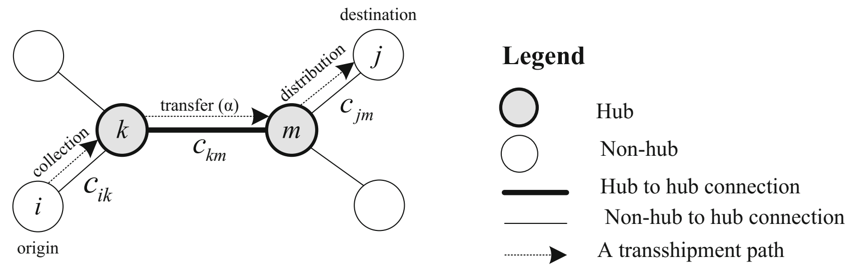

Fig. 1 Transshipment between pairs of non-hub nodes via hubs in single allocation

4.1.2 The deterministic model

As the first step of the proposed systematic procedure, a deterministic hub location model (DHLM), with regard to defined characteristics of the problem, should be derived. Therefore, using the previous sets of notations, the deterministic integer programming formulation of the single allocation p-hub median problem $\left(p\text{-hub}/D/\mathrm{SA}/\sum_{\text{flow}} + \sum_{\text{hub}}.\right)$ is presented based on O'Kelly [3], with little manipulation (adding the setup cost for opening hubs) as follows:

DHLM:

$$\text{Min} \sum_{i\in N}\sum_{j\in N} h_{ij}\left(\sum_{k\in N} c_{ik}y_{ik} + \alpha\sum_{k\in N}\sum_{m\in N} c_{km}y_{ik}y_{jm} + \sum_{m\in N} c_{jm}y_{jm}\right) + \sum_{k\in N} f_k x_k \tag{1}$$

$$\text{s.t.} \quad \sum_{k\in N} y_{ik} = 1 \quad \forall i\in N \tag{2}$$

$$\sum_{k\in N} x_k = p \tag{3}$$

$$y_{ik} \le x_k \quad \forall i,k\in N \tag{4}$$

$$y_{ik}\in\{0,1\} \quad \forall i,k\in N \tag{5}$$

$$x_k\in\{0,1\} \quad \forall k\in N. \tag{6}$$

The objective function consists of the transportation cost in forms of collection, transfer, and distribution as first, second, and third term, respectively, and the total setup cost for opening p-hub facilities. The unit transportation cost in transfer must be smaller than the collection and distribution cost, so $0\le\alpha\le 1$ and it is multiplied by the total cost of transshipment. Constraint (2) guaranties the single allocation of non-hub nodes to hubs. The exact number of required hubs is ensured by constraint (3). Constraint (4) states that the demand node i cannot be connected to a hub at j unless a hub at node j is opened, and constraints (5) and (6) are standard binary constraints.

4.1.3 Two-stage stochastic model

Considering the stochastic nature of hub location problems, these kinds of problems could well be defined in a stochastic environment and modeled as a two-stage stochastic programming problem. Usually, the most important uncertainty in this type of problems is the uncertainty in demand between the customers (i.e., the amount of flow between the nodes). Thus, adopting an appropriate approach to face this kind of uncertainty would have a significant impact in reducing the costs and improving the overall performance of the decision-making system. Therefore, in this stochastic model, it is assumed that the demand in future periods is uncertain and different realizations (scenarios) are possible for it. Moreover, in the real world, in addition to the cost of transportation between the hubs, there is a setup cost to open the hub facilities in potential nodes. Considering the fact that the decision about the location of the hub facilities is usually

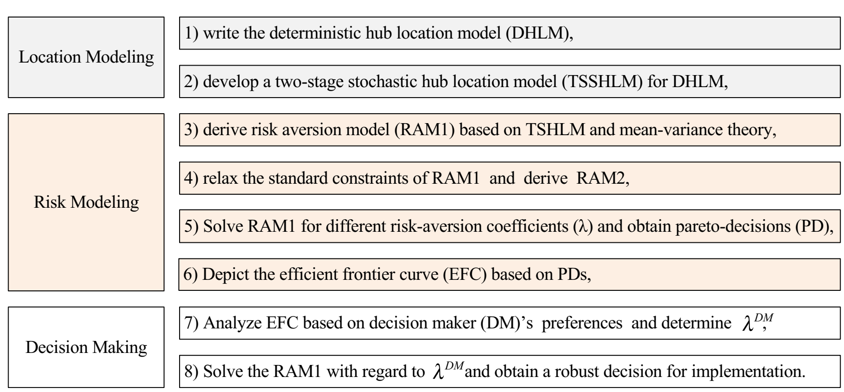

Fig. 2 The proposed decision-making procedure to make robust decisions for stochastic hub location problem

made at the beginning of the period before the occurrence of stochastic conditions, the problem is formulated as a two-stage stochastic hub location model (TSSHLM). In the first stage, location of the hub facilities is decided upon, and in the second stage, the suitable allocation of non-hub nodes to hub facilities is determined to meet the demand as shown in Fig. 3.

Considering ξ_{ij} to present the demand or flow between origin i and destination j as a random parameter, and ϕ as a realized certain scenario from a set of all possible scenarios Φ, then the general form of TSSHLM for p median, which includes two separate sub-models, first-stage model (FSM) and second-stage model (SSM), could be written as follows:

TSSPHL:

$$\min_x \quad F(x) + E_\xi[Q(x,\xi(\phi))] \tag{7}$$

where $F(x)$ represents the total setup cost of FSM, $E_\xi[.]$ denotes the mathematical expectation with respect to demand random vector ξ, and $E_\xi[Q(x,\xi)]$ is the *recourse function*, in which $Q(x,\xi)$ is the optimal value of SSM.

FSM:

$$\min_x \quad F(x)\sum_{k=1}^{n} f_k x_k \text{ s.t.}(3) \text{ and } (6) \tag{8}$$

SSM:

$$\min_y \sum_{\phi\in\phi}\sum_{i\in N}\sum_{j\in N}\xi_{ij}(\phi) \left(\sum_{k\in N} c_{ik}y_{ik}(\phi)+\alpha\sum_{k\in N}\sum_{m\in N} c_{km}y_{ik}(\phi)y_{im}(\xi)+\sum_{m\in N} c_{jm}y_{jm}\right) \tag{9}$$

$$\text{s.t.} \quad \sum_{k\in N} y_{jm}(\phi) = 1, \quad \forall i\in N; \forall \phi\in\Phi \tag{10}$$

$$y_{ik}(\phi)\le x_k, \quad \forall i,k\in N; \forall\, \phi\in\Phi \tag{11}$$

$$y_{ik}(\phi)\in\{0,1\} \quad \forall i,k.\in N; \forall\, \phi\in\Phi \tag{12}$$

where $y_{ik}(\phi)$ denotes the allocation of non-hub i to hub facility k for a realized scenario ϕ. The aim of this model is to minimize the expected value of the total cost. Since the cost function of the first stage is constant and independent of the stochastic parameters, it remains unchanged and only the cost function of the second stage should be derived. The expected value of the second-stage objective function, $E_\xi[Q(x,\xi(\phi))]$, is as follows:

RP:

$$\min_y \sum_{\phi\in\Phi}\sum_{i\in N}\sum_{j\in N} p(\phi)\xi_{ij}(\phi) \left(\sum_{k\in N} c_{ik}y_{ik}(\phi)+\alpha\sum_{k\in N}\sum_{m\in N} c_{km}y_{ik}(\phi)y_{jm}(\xi)+\sum_{m\in N} c_{jm}y_{jm}(\phi)\right) \quad \text{s.t.} \quad (10) \text{ and } (12). \tag{13}$$

where $p(\phi)$ is the probability of occurring for scenario ϕ.

4.2 Risk modeling

4.2.1 Mean-variance theory

In a risky situation, decision makers (DMs) exhibit three different behaviors in the forms of risk-seeking, risk-neutral, and risk-averse that originated from concavity, linearity, and convexity of their utility function, respectively, as shown in Fig. 4. It is obvious that a rational DM would want to choose a policy with a higher expected value and a lower variance of positive utilities, such as profit. In other words, it is reasonable that a DM have a risk-averse behavior to deal with a risky situation. Therefore, in this study, DM is a normal person who has a risk-averse behavior and tries to make more robust decisions with less risk.

The formulation of mean-variance theory is a basic optimization tool to model risk concept with regard to a quadratic utility function for DM's preferences. In this situation, a trade-off between risk and utility function using a risk aversion coefficient λ is modeled.

$$U(x) = x - \lambda x^2 \tag{14}$$

In the quadratic utility function in Eq. (14), risk-seeking, risk-neutral, and risk-averse behavior is adjusted based on the

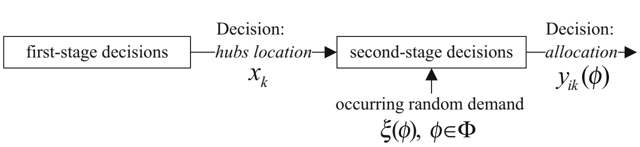

Fig. 3 The decision-making process for a two-stage stochastic *p*-hub location model

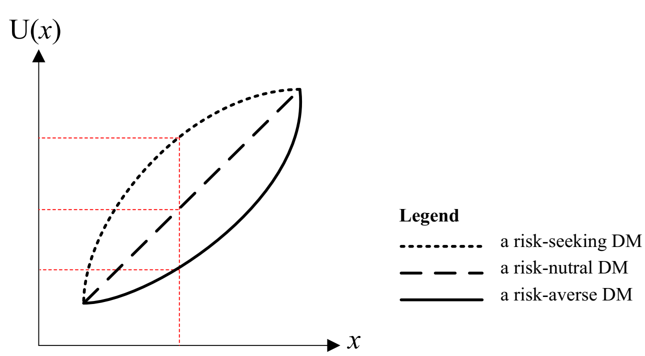

Fig. 4 The behavior of decision makers based on quadratic utility function

positive, zero, and negative value for risk aversion coefficient λ, respectively. Therefore, in the stochastic situation in this problem with random parameter ξ, the cost function $f(x,\xi)$ for a risk-averse DM can be derived based on the quadratic utility function (negative utility) as Eq. (15)

$$U(f(x,\xi)) = \min\left\{\mu_{f(x,\xi)} + \lambda\sigma^2_{f(x,\xi)}\middle|\lambda > 0\right\} \tag{15}$$

where μ and σ^2 represent the expected value and variance of the cost function $f(x,\xi)$, respectively.

4.2.2 Mean-variance risk model

Nowadays, the hubs play an indispensable role in the communicative environments in both industrial and service competitive sectors. It is obvious that the main concept of the hub location was introduced to make more economical and convenient decisions in the facility location area. Due to the fact that the majority of real location decisions are made in stochastic situations for a long period of time and changing the contemporary decisions is costly and irrational, it is necessary to have an especial tool to analyze the problem and make more robust and tough decisions. The traditional approach in stochastic hub location problems is to solve them without considering the range of volatility and the reliability of obtained solutions, while the nature of this type of problems is highly indeterminate. On the other hand, the adopted policy should be able to meet the system requirements for a long time, since changing the implemented initial decisions would be very costly. This clarifies the importance of obtaining solutions with higher reliability or lower risk in this type of problems. Actually, it has been proven that in many stochastic problems, only using the expected value approach cannot be a reliable decision criterion (see for example [18]).

These stated facts have motivated the authors to present a novel approach, considering risk management in the stochastic SAPHMP.

$$\mathrm{TC} = F(x) + Q(x,\xi) \tag{16}$$

where

$$F(x) = \sum_{j\in N} f_j x_j \tag{16 − 1}$$

and

$$Q(x,\xi) = \sum_{i\in N}\sum_{j\in N}\xi_{ij}\left(\sum_{k\in N} c_{ik}y_{ik} + \alpha\sum_{k\in N}\sum_{m\in N} c_{km}y_{ik}y_{jm} + \sum_{m\in N} c_{jm}y_{jm}\right). \tag{16 − 2}$$

To calculate the expected value and variance (first and second moment) of the total cost, the basic terms are introduced as Eqs. (17), (18), and (19).

$$E[Q(x,\xi)] = \sum_{\phi\in\Phi} p(\phi)Q(x,\xi(\phi)) \tag{17}$$

$$E[F(x))] = F(x) \tag{18}$$

$$E\left[Q^2(x,\xi)\right] = \sum_{\phi\in\Phi} p(\phi)Q^2(x,\xi(\phi)). \tag{19}$$

By applying the main concept of the first and second moment and substituting Eqs. (17) to (19), the expected value and variance of total cost are derived as Eq. (20) and (21), respectively.

$$\begin{aligned} E(TC) &= E[F(x) + Q(x,\xi)] = F(x) + E[Q(x,\xi)] \\ &= \sum_{j\in N} f_j x_j + \sum_{\phi\in\Phi} p(\phi)Q(x,\xi(\phi)) \end{aligned} \tag{20}$$

$$\begin{aligned} \mathrm{Var}(\mathrm{TC}) &= E\left[(\mathrm{TC}-E[\mathrm{TC}])^2\right] = E\left[(F(x) + Q(x,\xi)-F(x)-E[Q(x,\xi)])^2\right] \\ &= E\left[(Q(x,\xi)-E[Q(x,\xi)])^2\right] = E\left[(Q(x,\xi))^2-2E[Q(x,\xi)]Q(x,\xi) + (E[Q(x,\xi)])^2\right] \\ &= E\left[(Q(x,\xi))^2\right]-2\,(E[Q(x,\xi)])^2 + (E[Q(x,\xi)])^2 = E\left[(Q(x,\xi))^2\right]-(E[Q(x,\xi)])^2 \\ &= \sum_{\phi\in\Phi} p(\phi)Q^2(x,\xi(\phi))-\left(\sum_{\phi\in\Phi} p(\phi)Q(x,\xi(\phi))\right)^2 \end{aligned} \tag{21}$$

where $E(.)$ and Var(.) are the expected value and variance mathematical operators.

Therefore, the quadratic cost utility function based on mean-variance theory could be written as follows:

$$\mathrm{MV} = E(\mathrm{TC}) + \lambda \mathrm{Var}(\mathrm{TC}), \quad \lambda > 0 \tag{22}$$

where λ is the risk aversion coefficient for the mean-variance (MV) function that takes different values depending on the DM's preferences in dealing with risky conditions.

There are two major difficulties in optimizing the MV model. First, there is heterogeneity between the variance and the expected value of cost function. Greater values of variance compared to the average costs will lead to loss of sensitivity of the model in different scenarios, and it is observed that the importance of average costs will fade away. In order to reduce this deficiency, the objective function is normalized as suggested in multi-objective optimization literature. In this approach, at first, the two functions of E(TC) and Var(TC) are individually optimized in both minimum and maximum states and then the risk measure (RM) is defined as the objective function of risk modeling (RM), which must be minimized as follows:

RM1:

$$\begin{aligned}&\mathrm{MinRM} = \left(\frac{E(\mathrm{TC})-E^*_{\min}(\mathrm{TC})}{E^*_{\max}(\mathrm{TC})-E^*_{\min}(\mathrm{TC})}\right) + \lambda\left(\frac{\mathrm{Var}(\mathrm{TC})-\mathrm{Var}^*_{\min}(\mathrm{TC})}{\mathrm{Var}^*_{\max}(\mathrm{TC})-\mathrm{Var}^*_{\min}(\mathrm{TC})}\right)\\ &\text{s.t.} \quad (2)-(6).\end{aligned} \tag{23}$$

The second problem is the mixed integer nonlinear (MINL) form of the RM1, which should be solved many times for different risk aversion coefficients, and the computational complexity makes it impossible to solve large-scale problems in a reasonable time. To overcome this problem and obtain the efficient frontier curve, a new procedure is presented as one of the innovative aspects of this study. The efficient frontier curve involves the efficient combinations of $(\lambda_i, \mu_i, \sigma_i)$, which is also called Pareto solutions. In order to obtain these Pareto solutions, RM2 could be solved in which the standard constraints from integer feasible space ($x_j, y_{ij} \in \{0,1\}$) are relaxed into continuous feasible space ($0 \le x_j, y_{ij} \le 1$) such as follows:

RM2:

$$\begin{aligned}&\mathrm{Min} \quad \mathrm{RM} = \left(\frac{E(\mathrm{TC})-E^*_{\min}(\mathrm{TC})}{E^*_{\max}(\mathrm{TC})-E^*_{\min}(\mathrm{TC})}\right) + \lambda\left(\frac{\mathrm{Var}(\mathrm{TC})-\mathrm{Var}^*_{\min}(\mathrm{TC})}{\mathrm{Var}^*_{\max}(\mathrm{TC})-\mathrm{Var}^*_{\min}(\mathrm{TC})}\right)\\ &\text{s.t.} \quad (2)\text{–}(4)\end{aligned} \tag{24}$$

$$0 \le x_k \le 1, \quad \forall k \in N \tag{25}$$

$$0 \le y_{ik}\ (\phi) \le 1, \quad \forall i,\ k \in N. \tag{26}$$

This research shows that the standard deviation and expected value of TC behave in the same way for similar risk aversion coefficients in the RM1 (mixed integer nonlinear programming) and RM2 (the relaxed mixed integer nonlinear programming). In order to verify this claim, a small-scale case study is provided with four nodes, two hubs, and two stochastic scenarios in both forms for different risk aversion coefficients.

Figure 5a, b shows the expected value of the TC and its standard deviation for different aversion coefficient values for both RM1 and RM2 formats, respectively. As anticipated, the rising trend for expected value and the falling trend for standard deviation change in an approximately similar pattern in both formats.

Figure 6 shows the change ratio of expected value versus standard deviation in the two formats, which does not demonstrate a significant difference for analyzing the risk and getting the suitable risk aversion coefficient. As a result, in real-world problems with larger scale, one can study the RM2 to analyze the risk and find the proper risk aversion coefficient, which is easier to solve and less complicated.

Therefore, to determine DM's preferences, it is necessary to depict the efficient frontier curve of RM2 in a three-dimensional diagram and ask the DM to assign an acceptable value for the risk aversion coefficient, considering reasonable risk and cost levels. For example, efficient frontier curve of both RM1 and RM2 has been shown in Fig. 6. Furthermore, the Pareto solutions are represented as Eq. (27).

$$\mathrm{PD} = \left\{ \left(\lambda^{(j)}, \mu^{(j)}_{\mathrm{TC}}, \sigma^{(j)}_{\mathrm{TC}}\right) \middle| \min\ \mathrm{RM2},\ \ \forall\, j = 1, 2, \ldots, 21;\ \lambda = \{0, 0.5, 1, 1.5, \ldots,\ \ 10\} \right\} \tag{27}$$

4.2.3 Determining risk aversion coefficient

The most important part of the risk analysis process is deciding about the risk aversion coefficient. Due to the fact that the p-hub median location problem is NP hard [24], solving RM1, which is a mixed integer nonlinear program, is more difficult, especially in large-scale problems. To overcome this issue, the relaxed mixed integer nonlinear programming, called RM2, is presented to determine risk aversion coefficient. In other words, based on mean-variance approach, an efficient trade-off between the expected value and the standard deviation should be found, which involves the DM's preferences. The

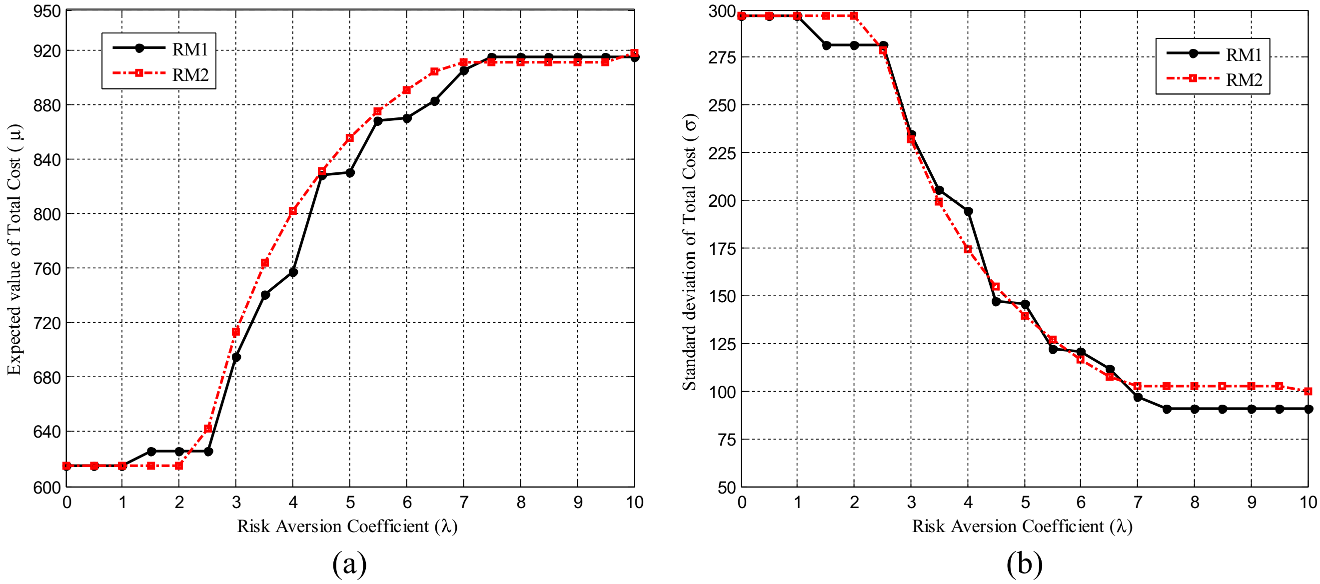

Fig. 5 The expected value (**a**) and standard deviation (**b**) of TC for different risk aversion coefficients for RM1 and RM2

risk-averse decision makers like to make decision with lower expected value and standard deviation of TC. Hence, in the mean-variance approach, a multi-objective model must be optimized, in which two heterogeneous objective functions E(TC) and Var(TC) should be minimized based on weighted sum method. The weighted sum method is one of the most popular methods to solve a multi-objective optimization problem, in which multiple objectives $f_i(x)$ are combined into a single objective $F(x)$, as follows:

$$\min \quad F(x) = \sum_{i=1}^{n} \gamma_i f_i(x) \tag{28}$$

where γ_i is a coefficient related to objective function $f_i(x)$. The scalar concept of "optimality" is not directly valid in the multi-objective optimization. In this situation, the notion of Pareto optimality has to be introduced. If S is supposed to be as a set of constraints of multi-objective problem, then a vector $x^* \in S$ is said to be the Pareto optimal for a multi-objective problem if all other vectors $x \in S$ have a higher value for at least one of the objective functions f_i, with $i = \{1, 2, \ldots, n\}$ or have the same value for all the objective functions.

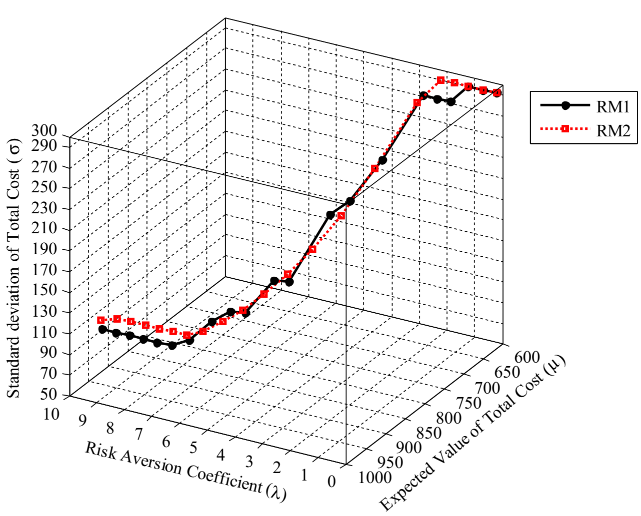

Fig. 6 The efficient frontier curve for RM1 and RM2

Due to the multi-objective optimization nature of mean-variance model, a Pareto optimal decision must be obtained. However, there are countless solutions and all of them cannot be obtained. Thus, a discrete interval from 0 to 10 with a step of 0.5 is considered for risk aversion coefficient and the relative solutions for each point are calculated. Based on the 21 Pareto solutions, the efficient frontier curve is formed using a scatter plot of these points and by fitting a quadratic curve, in which all the possible Pareto optimal decisions will be estimated. This curve includes all the different preferences of the DMs. In other words, there will be a risk aversion coefficient for every preference of the decision maker (μ,σ) and each risk-averse coefficient corresponds to a certain preference and then the efficient frontier curve consists of all the risk-averse DMs' preferences. The value of risk aversion coefficient is

Table 1 The information of the case study with ten nodes and five likely scenarios for demand

Nodes (N)	Hubs (p)	Scenarios (Φ)	Probability of scenarios ($p(\phi)$)	Discount factor (α)
Shiraz, Tehran	p=3	Very low	0.11	0.6
Kerman, Semnan		Low	0.22	
Kermanshah, Tabriz		Middle	0.33	
Borujerd, Mashhad		High	0.22	
Kashan, Arak		Very high	0.12	

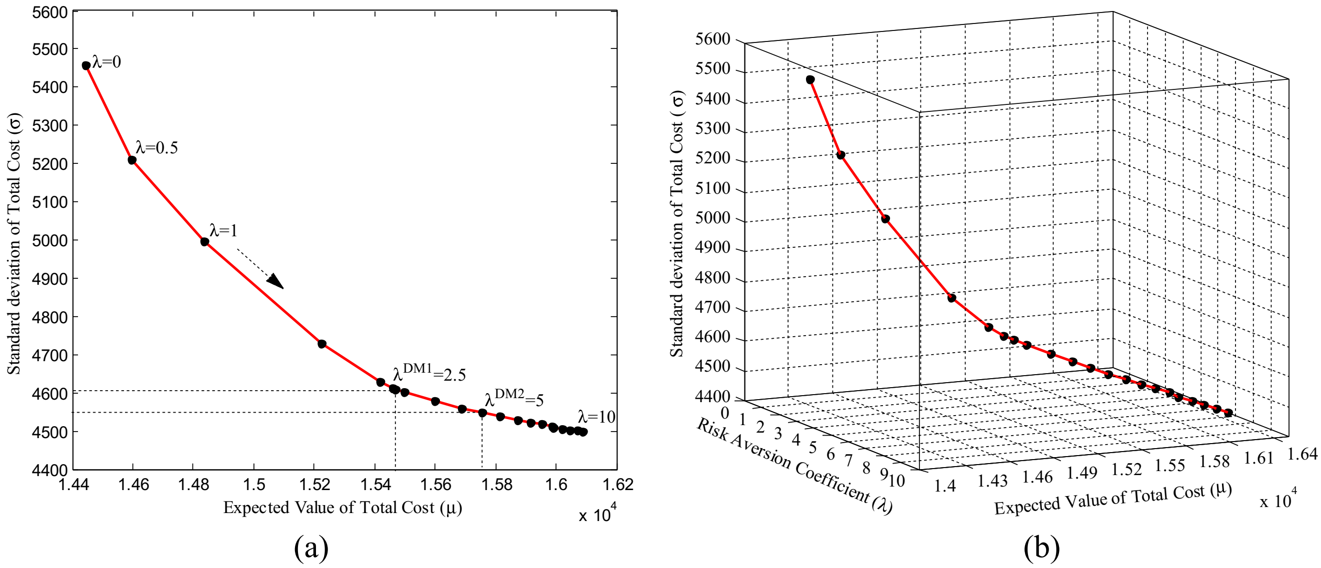

Fig. 7 Two-dimensional (**a**) and three-dimensional (**b**) demonstration of Pareto solutions and the efficient frontier curve for different DMs' preferences

chosen based on the DM's preferences and his/her risk attitude, and it may be different for each DM. Therefore, DMs should be able to select a risk aversion coefficient on the efficient frontier curve based on their preference to obtain a reliable decision for implementation.

5 A case study

In order to implement the systematic approach and illustrate the proposed procedures, a network of automobile part distribution and transportation system is considered. A nationwide corporation in Iran has decided to establish a new system of distribution of automobile parts among several subsidiaries around the country. For this, a stochastic location-allocation hub problem is considered. Due to the strategic and long-term nature of decisions, the reliability of the policy is critical for the corporation so it is necessary to consider a risk-based approach.

It is required to establish three hub centers among the cities where subcontracts are located and a policy for transshipments between cities in order to realize future demands. Based on historical data and consultation with experts, five levels of demands are separated as future scenarios. The basic data for the problem is mentioned in Table 1.

The problem is implemented in General Algebraic Modeling System (GAMS) modeling environment using the GAMS/CONOPT and GAMS/DICOPT solvers for RM2 and RM1 models, respectively. All tests were executed on a personal computer equipped with a 4.2-GHz Intel Pentium 4 CPU and a 2-GB RAM, based on Windows platform.

In order to obtain the Pareto solutions and depict the efficient frontier curve, the RM2 was solved with 21 different risk aversion coefficients of $\lambda=\{0, 0.5, 1,\ldots, 10\}$. The relevant frontier curve is shown in Fig. 7a, b. As expected, increasing a risk-averse level of the DM leads to an increase in the expected value of total cost and a decrease in the standard deviation. Furthermore, it is obvious that there are unique decisions for each level of risk aversion coefficient.

In this case study, two representatives are nominated to discuss the issue, DM1 and DM2. DM1, the representative of the chief executive officer, has a preference of $\lambda^{DM1}=2.5$, while DM2, the representative of the subcontractors, tends to accept lower risk levels and makes the decisions which represent a risk aversion coefficient of $\lambda^{DM2}=5$. In other words, the DMs select their risk aversion coefficients based on their preferences for the expected value and standard deviation of total cost from the frontier curve.

The proposed approach is expected to show different results based on different attitudes of the DMs. In order to obtain

Table 2 The location-allocation results of the case study according to DMs' preferences

Decision maker	λ^{DM}	3-hub	Allocation	μ_{TC}	σ_{TC}
DM1	2.5	Tehran	Tabriz, Kermanshah	15,461.08	4613.19
		Kerman	Semnan, Shiraz, Mashhad		
		Kashan	Arak, Borujerd		
DM2	5	Tehran	Tabriz, Semnan	15,753.98	4547.83
		Kerman	Kashan, Shiraz, Mashhad		
		Kermanshah	Arak, Borujerd		

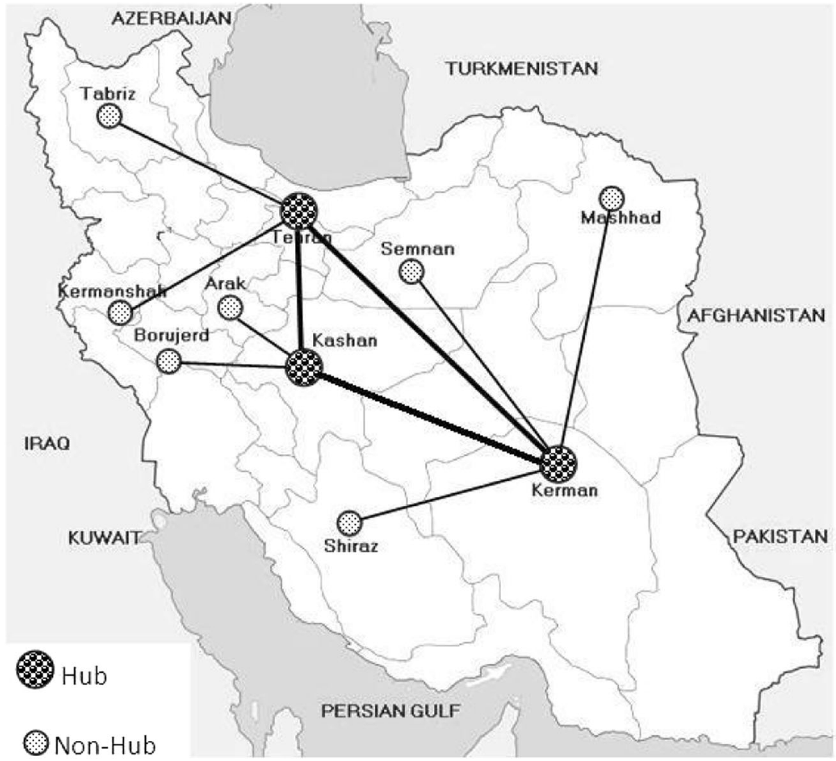

Fig. 8 Optimal hub location and allocation policy for DM1

DMs' policies, the RM1 optimization is run with regard to the two risk aversion coefficients, separately. As shown in Table 2, the two robust optimal policies consisting of location of hubs, allocation of non-hubs to hubs, and the expected value and standard deviation of the total cost are derived.

Based on the information in Table 2 and the location of each subcontractor on Iran's map, the robust optimal network configuration for this distribution system is depicted for DM1 and DM2's preferences in Figs. 8 and 9, respectively.

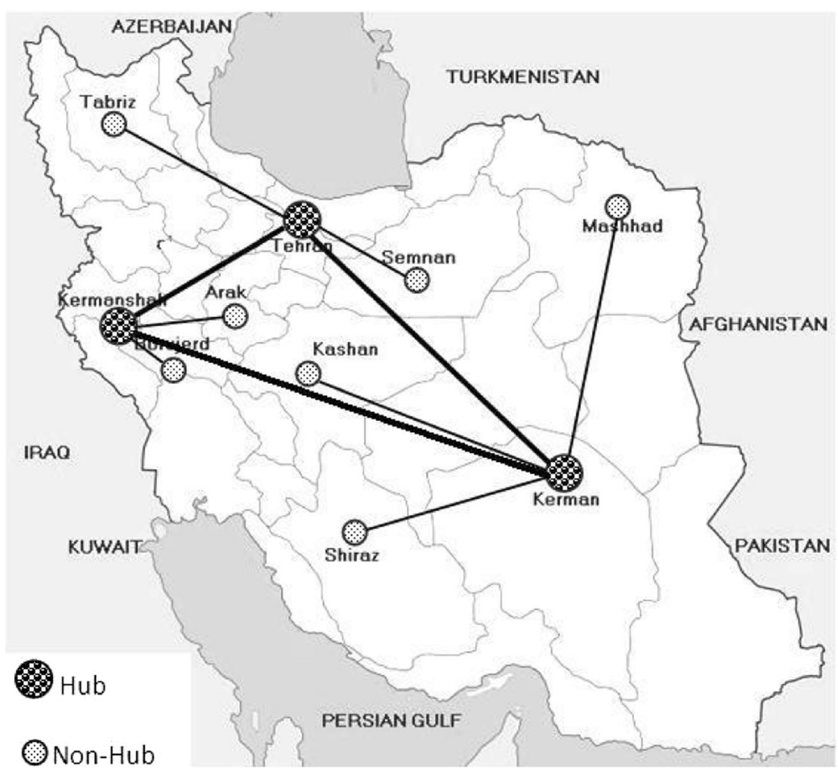

Fig. 9 Optimal hub location and allocation policy for DM2

6 Conclusion and future research directions

A critical concern of today's risk-averse decision makers is managing the uncertain conditions, considering reliability of decisions. In hub location-allocation problems, there has been no significant attempt to study the problem in this situation; even stochastic models of this problem without considering risk are quiet rare in the literature. This issue led the authors to develop a systematic risk management approach to make robust decisions according to DMs' preferences for a location-allocation *p*-hub median problem for the first time. This approach, built on mean-variance theory and two-stage stochastic programming concepts, is comprised of three main phases, namely location modeling, risk modeling, and decision making, each of which phase has several steps. The first phase explains how to develop a proper two-stage stochastic model, based on the related deterministic model. In the second phase, DM's utility function is derived based on total cost function in the form of mean-variance. One of the main problems associated with this measure is heterogeneity between the variance and the expected value of costs in the objective function. In order to eliminate the effect of this heterogeneity, a normalization method was suggested, which is of huge assistance in getting reasonable results. Then, an innovative procedure was proposed in order to enhance the calculation efficiency especially for large-scale cases. Obtaining Pareto solutions, depicting frontier curve, determining the DM's risk aversion coefficient, and obtaining the robust policy through optimization based on the DM's preference are conducted in the third phase. Finally, a practical case study of a distribution system of automobile parts in Iran demonstrated the effectiveness of the proposed approach.

The authors hope that this paper could be a proper commencement for including other risk measures such as VAR and CVAR in other location problems. This work may be extended in several directions. From a modeling perspective, newer hub location problems such as multiple allocation *p*-hub median problem, *p*-hub center problem, and emergency facility location can be studied. Also from the risk management viewpoint, there is a very broad area for future research. Various risk measures with different advantages and disadvantages have been introduced in the literature of financial management, which can now be studied in the field of hub location problems. Finally, about the solution methods, since the problem is NP hard on its own and real-world risk problems need to be solved in greater dimensions, the development of exact or effective heuristic or meta-heuristic methods seems to be necessary for the future cases.

References

1. Markowitz H (1959) Portfolio selection: efficient diversification of investments. Cowles Foundation, New Haven, p 94
2. O'Kelly ME (1986) Activity levels at hub facilities in interacting networks. Geogr Anal 18(4):343–356
3. O'Kelly ME (1987) A quadratic integer program for the location of interacting hub facilities. Eur J Oper Res 32(3):393–404
4. Campbell JF, Ernst AT, Krishnamoorthy M (2002) Hub location problems. Facil Locat Appl Theory 1:373–407
5. Alumur S, Kara BY (2008) Network hub location problems: the state of the art. Eur J Oper Res 190(1):1–21
6. Farahani RZ, Hekmatfar M, Arabani AB, Nikbakhsh E (2013) Hub location problems: a review of models, classification, solution techniques, and applications. Comput Ind Eng 64(4):1096–1109
7. Louveaux F (1986) Discrete stochastic location models. Ann Oper Res 6(2):21–34
8. Louveaux FV, Peeters D (1992) A dual-based procedure for stochastic facility location. Oper Res 40(3):564–573
9. Laporte G, Louveaux FV, van Hamme L (1994) Exact solution to a location problem with stochastic demands. Transp Sci 28(2):95–103
10. Snyder LV (2006) Facility location under uncertainty: a review. IIE Trans 38(7):547–564
11. Marianov V, Serra D (2003) Location models for airline hubs behaving as M/D/*c* queues. Comput Oper Res 30(7):983–1003
12. Mohammadi M, Jolai F, Rostami H (2011) An M/M/c queue model for hub covering location problem. Math Comput Model 54(11): 2623–2638
13. Sim T, Lowe TJ, Thomas BW (2009) The stochastic p-hub center problem with service-level constraints. Comput Oper Res 36(12): 3166–3177
14. Yang TH (2009) Stochastic air freight hub location and flight routes planning. Appl Math Model 33(12):4424–4430
15. Contreras I, Cordeau JF, Laporte G (2011) Stochastic uncapacitated hub location. Eur J Oper Res 212(3):518–528
16. Hult E, Jiang H, Ralph D (2011) Exact computational approaches to a stochastic uncapacitated single allocation p-hub center problem. Comput Optim Appl 1–16
17. Alumur SA, Nickel S, Saldanha-da-Gama F (2012) Hub location under uncertainty. Transp Res B Methodol 46(4):529–543
18. Bradley JV (1975) Probability, decision, statistics. Prentice-Hall, Englewood Cliffs
19. Wagner MR, Bhadury J, Peng S (2009) Risk management in uncapacitated facility location models with random demands. Comput Oper Res 36(4):1002–1011
20. Wang S, Watada J (2012) A hybrid modified PSO approach to VaR-based facility location problems with variable capacity in fuzzy random uncertainty. Inf Sci 192:3–18
21. Azad N, Davoudpour H (2010) A two echelon location-routing model with considering value-at-risk measure. Int J Manag Sci Eng Manag 5(3):235–240
22. Zhai H, Liu Y, Chen W (2012) Applying minimum-risk criterion to stochastic hub location problems. Procedia Eng 29:2313–2321
23. Mohammadi M, Jolai F, Tavakkoli-Moghaddam R (2013) Solving a new stochastic multi-mode *p*-hub covering location problem considering risk by a novel multi-objective algorithm. Appl Math Model 37(24):10053–10073
24. Kara BY, Tansel BC (2000) On the single-assignment *p*-hub center problem. Eur J Oper Res 125(3):648–655